שאלה 1:

א. ePub listening, a new approach:

ב. ePub listening, a new approach:

ג. ePub listening, a new approach:

ד. ePub listening, a new approach:

ה. ePub listening, a new approach:


\[ w_1(t) = x^2(t) = A^2 \cos^2(40\pi t) = \frac{A^2}{2} \left(1 + \cos(80\pi t)\right) \]

\[ W_1^F(\omega) = \frac{\pi A^2}{2} \left[ \delta(\omega + 80\pi) + \delta(\omega - 80\pi) + 2\delta(\omega) \right] \]

\[ W_1^F(\theta) = \frac{\pi A^2}{2T_s} \left[ \delta(\theta + 0.4\pi) + \delta(\theta - 0.4\pi) + 2\delta(\theta) \right] \]

\[ y_1(t) = \frac{A^2}{2} \left[1 + \cos(20\pi t)\right] \]
א. המשוואה לעדכון מקדמי מסנן המסתגל נתונה על ידי:

\[ \hat{w}[n+1] = \hat{w}[n] - \frac{1}{2} \mu \nabla J[n] \]

נחשב את הגрадיאנט (הנגזרת ביחס למקדמי המסנן) של פונקציית מחיר הנתונה בשאלה:

\[ \nabla J[n] = \frac{\partial}{\partial \hat{w}^T[n]} \left( d[n] - \hat{w}^T[n]y[n] \right)^2 + \alpha \frac{\partial \left( \hat{w}^T[n] \hat{w}[n] \right)}{\partial \hat{w}^T[n]} \]

\[ = -2d[n]e[n] + 2\alpha \hat{w}[n] \]

נציב במשוואה העדכון:

\[ \hat{w}[n+1] = \hat{w}[n] - \frac{1}{2} \mu \left( -2d[n]e[n] + 2\alpha \hat{w}[n] \right) \]

\[ = \hat{w}[n] + \mu y[n]e[n] - \mu \alpha \hat{w}[n] \]

\[ = (1 - \mu \alpha) \hat{w}[n] + \mu y[n]e[n] \]

ב. עבור \( \alpha = 0 \), נקבל את המשוואה האלגוריתם שלמסנן LMS.

בשלי הלוגריה, \( \alpha = 1/\mu \).

עבור \( \mu \ll 1 \), התמקם המסתגל שלול בפונקציה המרを持ך.

\[ J[n] = |e[n]|^2 + \frac{1}{\mu} \| \hat{w}[n] \|^2 \approx \frac{1}{\mu} \| \hat{w}[n] \|^2 \]

בנוסף, מכיוון שהמשוואה העדכון הוא משולב בתוכלו, עבור \( \mu \ll 1 \), ניתן להрогו "קטו" הקבילה:"联合国"}

כבר \( \mu \approx 1 \) נשים ת موقف פונקציה המר, בן, למסנן קפע.
שאלה 3:

(a) $F_{\text{Nyquist}} = 12 \text{ [kHz]}$ (1)

$$F_{\text{min}} = 4 \text{ [kHz]}$$ (2)

(b) $F_{\text{Nyquist}} = 12 \text{ [kHz]}$ (1)

The minimum frequency is $F_{\text{min}} = 4 \text{ [kHz]}$.

For the ideal LP (low-pass) filter, the order of the filter is determined by the maximum passband.

If we sample at a frequency equal to the Nyquist frequency, there are no distortions, and therefore the ratio between the sampling frequency and the passband is:

$$\Delta \Theta = \frac{\Delta f}{F_s} = \frac{2\pi}{F_s}$$

Which is a monotonically decreasing function of $sF$ and reaches its maximum for $F_s = F_{\text{Nyquist}}$. In our case ($F_s = F_{\text{Nyquist}}$).

Therefore, the ideal finite impulse response (FIR) filter cannot be implemented by windowing the ideal.

The frequency spectrum of the signal is:

$X_s^F(f)$

1 4 5 6

$f$ [kHz]
The required filter is LP

\[ A_p = 0.1 \text{dB} = -20 \log(1 - \delta_p) \Rightarrow \delta_p = 0.0114 \]

\[ A_s = 60 \text{dB} = -20 \log(\delta_s) \Rightarrow \delta_s = 10^{-3} \]

\[ A_s = 60 \Rightarrow \text{Blackman} \]

\[ \frac{12\pi}{L} \leq \Delta \theta = |\theta_p - \theta_s| = \frac{4\pi}{9} \]

\[ \frac{12\pi}{N+1} \leq \frac{4\pi}{9} \Rightarrow N \geq 26 \]

\[ N = 26 \text{ is the minimum odd value of } N, \text{Type I, II} \text{-LP} \]
4. Convolution is cyclically equivalent to linear convolution when the sequence length is an integer multiple of N. This means that if we have two sequences:

\[ u[n] = \tilde{x}[n] \ast y[n] = \sum_{k=-\infty}^{\infty} y[k] \tilde{x}[n-k] = \sum_{k=-\infty}^{\infty} y[k] x[(n-k) \mod N] = \sum_{k=0}^{N-1} y[k] x[(n-k) \mod N] \]

Then we have:

\[ u[n] = z[n] \]

Where \( z[n] \) is the output of the linear convolution.

5. For the DTFT, we have:

\[ X^d[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi nk/N} = \sum_{n=0}^{N-1} \tilde{x}[n] e^{-j2\pi n/N} \]

\[ = \sum_{n=0}^{N-1} a_k e^{-j2\pi n/N} e^{-j2\pi n/N} \]

\[ = \sum_{k=0}^{N-1} a_k \sum_{n=0}^{N-1} e^{-j2\pi n/N} e^{-j2\pi n/N} \]

\[ = N \sum_{k=0}^{N-1} a_k \delta[(k-k') \mod N] = Na_k \]

And:

\[ X^f(\theta) = \sum_{n=-\infty}^{\infty} \tilde{x}[n] e^{-j\theta n} \]

\[ = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{N-1} a_k e^{-j2\pi nk/N} e^{-j\theta n} \]

\[ = \sum_{k=0}^{N-1} a_k \sum_{n=-\infty}^{\infty} e^{-j2\pi n/N} e^{-j\theta n} \]

\[ = 2\pi N \sum_{k=0}^{N-1} X^d[k] \delta\left(\theta - \frac{2\pi}{N} k\right) \]

The DTFT of \( e^{j\theta n} \) is:

\[ DTFT\{e^{j\theta n}\}(\theta) = 2\pi \delta(\theta - \theta_0) \]
The relationship does not hold.

Reason:

The values of $D_T$ at points $2k\frac{L}{\pi}$ are zero because these points do not form components of period $N\frac{L}{\pi}$ in the periodic repetition of the signal in its definition.

Let us clarify the relationship of $\hat{d}_{LXk}$ to the original character:

$\hat{d}_{LXk}$ are samples of a single-period sequence of the character $2k\frac{L}{\pi}$, which means

\[
\hat{\{W\}}_{k2} = \frac{1}{2\pi} \sum_{k=0}^{N-1} a_k \delta\left(\theta - \frac{2\pi}{N} k\right), -\pi \leq \theta \leq \pi
\]

If we use the relationship below

\[
\hat{d}_{LXk} = \frac{2\pi}{L} \sum_{k=0}^{N-1} W_k \left(\frac{2\pi}{L} - \frac{2\pi}{N} k\right)
\]

where the signal $X_{d}[k]$ is the convolution with a rectangular window of length $N$, we can state that the relationship holds only if $k = mN$.

Note: Even if the character is a whole multiple of $L$, the relationship does not hold for $k = mN$.