6

\textbf{Problem 1}

As stated in the lectures, the update formula is given by:
\[
\hat{w}(n+1) = \hat{w}(n) - \frac{1}{2} \mu \nabla J_s(n) \tag{1}
\]

(Hint: horizon \( \frac{1}{2} \) to choose the best metric for the specific function and the gradient of the function.

The gradient - the gradient of the cost function is:

\[
\nabla J_s(n) = \frac{\partial J_s(n)}{\partial \hat{w}''(n)} = \frac{\partial \left( |e(n)|^2 \right)}{\partial \hat{w}''(n)} = 2|e(n)|^2 \frac{\partial \left( |e(n)|^2 \right)}{\partial \hat{w}''(n)}
\]

As we wish to minimize the cost function.

Note: The constant \( u \) can be inserted as:
\[
\frac{\partial \left( |e(n)|^2 \right)}{\partial \hat{w}''(n)} = -2u[n]e(n)
\]

Moreover, we get:
\[
\hat{w}(n+1) = \hat{w}(n) - \frac{1}{2} \mu \frac{\partial \left( |e(n)|^2 \right)}{\partial \hat{w}''(n)} = \hat{w}(n) + 2\mu u[n]e(n)\hat{e}(n) = \hat{w}(n) + \mu u[n]e(n)\hat{e}(n)
\]

As we wish to change the constant and the horizon (we choose a horizon and \( \mu \) - constant)

and:

\textbf{Problem 2}

By solving the following:
\[
\varepsilon(n) = w_0 - \hat{w}(n)
\]
\[
e_0(n) = d(n) - \hat{w}''(n)u(n)
\]
\[
e(n) = d(n) - \hat{w}''(n)u(n)
\]
\[
\hat{w}(n+1) = \hat{w}(n) + \mu u[n]e(n)
\]

Formulations of the equations:
\[
v(n+1) = v(n) + \mu v(n)
\]
ראשית נفتح את הביטוי עבור \( \varepsilon(n+1) \):

\[
\varepsilon(n+1) = w_0 - \hat{w}(n+1) = w_0 - \left[ \hat{w}(n) + \mu u[n] (d(n) - \hat{w}''(n) u(n)) \right]
\]

\[
= w_0 - \hat{w}(n) - \mu u[n] d(n) + \mu u[n] u''(n) \hat{w}(n)
\]

נסדר איברים ונוסיף ונפחית את הביטוי

\[
\varepsilon(n+1) = w_0 - \hat{w}(n) - \mu u[n] u''(n)w_0 + \mu u[n] u''(n) \hat{w}(n) - \mu u[n] d(n) + \mu u[n] u''(n) w_0
\]

\[
= (I - \mu u[n] u''[n]) (w_0 - \hat{w}(n)) - \mu u[n] (d(n) - u''[n] w_0)
\]

\[
= (I - \mu u[n] u''[n]) \varepsilon(n) - \mu u[n] \bar{c}_0(n)
\]

**שאלה 3**

משוואה לעכון מקדמים המסהflate מתונה על ידי:

\[
\ddot{w}[n+1] = \dot{w}[n] - \frac{1}{2} \mu \nabla J[n]
\]

נחשב את הנורדיון של הנגזרת הבוית של מקדמים המסהflate нельзя מתון בשאלה

\[
\nabla J[n] = \frac{\partial}{\partial \ddot{w}[n]} \left( d[n] - \dot{w}^T[n] u[n] \right)^2 + \alpha \frac{\partial \dot{w}^T[n] \ddot{w}[n]}{\partial \ddot{w}[n]}
\]

\[
= -2u[n] e[n] + 2\alpha \ddot{w}[n]
\]

\[
\dot{w}[n+1] = \dot{w}[n] - \frac{1}{2} \mu (-2u[n] e[n] + 2\alpha \ddot{w}[n])
\]

\[
= \dot{w}[n] + \mu u[n] e[n] - \mu \alpha \ddot{w}[n]
\]

\[
= (1 - \mu \alpha) \dot{w}[n] + \mu u[n] e[n]
\]