ארוגות אקריאים - 044202
בון 1 - מועד ב'

הגיוון
1.مشך הבון - שעונית.
2./power 3 שאלות. בידקה מברשות 6 שניות לכל שיר.
3.אף לא התחממות בוחם עירetically שורר בזירה.
4.אין להתחמם את מחברת ההחיה בולדה.
5.כיתוב זכה ברוך.
6.שותבוך לא מושמע או触摸ב ונחבלה.
7.החות ממנה בלשון זכר מחוות נוחה בולדה.
8.אתו זרא שחרותה פוד שופטת על מחברת ההחיה.

שם לעבל: בכלי צהוב נחתה חתירה במזרח,埃尔 أو דריה/ת, וכלכל נגורדה את. ואת בתנאי שליא נכתב
שומ עבד זרח בraphו כליה/ת.

בצלחה!
(30%) 1

The answers are written in Hebrew. The next steps are:

1. When $Y$ is Pois $(x), X = x$ then $Y$ is a mixture of $X$. $Y$ follows an exponential distribution. MMSE estimation:

$$\int_0^\infty t^k e^{-xt} \, dt = \frac{k!}{x^{k+1}}, x > 0$$

and the estimator is: 


2. When $Y$ is a mixture of $X, N, N_1, N_2, \ldots, N_k$ (10%).

$$Y = X + N$$

$$Z_1 = N + N_1$$

$$Z_2 = N_1 + N_2$$

$$\vdots$$

$$Z_k = N_{k-1} + N_k$$

$$Z_{k+1} = N_k$$

The results of the estimators are:

$$\hat{X}_1 = \mathbb{E} [X \mid Y, Z_1, Z_2, \ldots, Z_{k+1}]$$

$$\hat{X}_2 = \mathbb{E} [X \mid Y, Z_2, \ldots, Z_{k+1}]$$

$$\hat{X}_3 = \mathbb{E} [X \mid Y, Z_1]$$

The estimation of the estimator is:

$$\hat{Y}_{\text{opt}}(X) = \mathbb{E} [Y \mid X] = X$$
\[
E[X \mid Y = k] = \int_0^\infty \alpha f_{X \mid Y}(\alpha \mid k) \, d\alpha
\]
\[
= \int_0^\infty \alpha \mathbb{P}\{Y = k \mid X = \alpha\} f_X(\alpha) \, d\alpha
\]
\[
= \int_0^\infty \alpha e^{-\alpha k} \mu e^{-\alpha \mu} \, d\alpha
\]
\[
= \frac{\int_0^\infty \alpha^k (\mu + 1)e^{-\alpha(\mu+1)} \, d\alpha}{\int_0^\infty \alpha^k \mu^k \, d\alpha}
\]
\[
= \frac{(k + 1)!/(\mu + 1)^{k+1}}{k!/(\mu + 1)^k}
\]
\[
= \frac{k + 1}{\mu + 1}
\]
\[
\hat{X}_{\text{opt}}(Y) = E[X \mid Y] = \frac{Y + 1}{\mu + 1}
\]
\[ Z = |X| \text{sign}(Y) \]

where \( X \) and \( Y \) are independent random variables following a standard normal distribution \( \mathcal{N}(0, 1) \).

\[ Z \sim \mathcal{N}(0, 1) \]

1. If \( z \geq 0 \), the distribution of \( Z \) is
\[ F_Z(z) = \frac{1}{2} \mathbb{P}\{Z \leq z\} + \frac{1}{2} \mathbb{P}\{Z \leq 0\} = \frac{1}{2} \mathbb{P}\{|X| \leq z\} + \frac{1}{2} \mathbb{P}\{-|X| \leq z\} \]

2. If \( z < 0 \), the distribution of \( Z \) is
\[ F_Z(z) = \frac{1}{2} \mathbb{P}\{-|X| \leq z\} = \frac{1}{2} \mathbb{P}\{|X| \geq -z\} = \frac{1}{2} \mathbb{P}\{X \geq -z\} + \frac{1}{2} \mathbb{P}\{X \leq z\} = \mathbb{P}\{X \leq z\} \]

\[ E[Z | Y] = E[|X| \text{sign}(Y) | Y] = \text{sign}(Y)E[|X| | Y] = \text{sign}(Y)E[|X|] = \frac{\sqrt{2}}{\pi} \text{sign}(Y) \]

The above results hold for both cases of \( Y \) and \( \text{sign}(Y) \) independent and dependent variables.
From (40%) of the total number of words, the following question is asked:

1. Which of the following statements are true?
   - (5%) of the total number of words.
   - (5%) of the total number of words.
   - (5%) of the total number of words.

2. What are the definitions of the following sets:
   - \( \{X_n\} \)
   - \( \{Z_n, n \geq 0\} \)
   - \( \{Y_n, n \geq 0\} \)

3. Define the following sets:
   - \( \{V_n - Y_n\} \)
   - \( \{V_n\} \)
   - \( \{Z_n\} \)

4. Prove the following statement:
   - (10%) of the total number of words.
   - (10%) of the total number of words.
   - (10%) of the total number of words.

5. Prove the following statement:
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   - (10%) of the total number of words.

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60. Prove the following statement:
    - (10%) of the total number of words.
ushedala lε'vemot. נְתַן: $b$

$$\mathbb{P} \{ V_4 = 2 \mid V_3 = 1, V_2 = 0, V_1 = 0 \} = \frac{1}{2}$$

שים מַהֲטֵה הָנֵי מִקְבֶּלֶת כֶּנֶּסי כִּי $Y_3 = 1$, $X_3 = 1$, $X_2 = 1$, $X_1 = -1$ שׁוּם וַאֲנָה.

$$\mathbb{P} \{ V_4 = 2 \mid V_3 = 1 \} < \frac{1}{2}$$

שים לֹא מְאֹד הָנֵי הָרֹקֶחֶת $\{ Y_3 = -1 \} - \{ Y_3 = 1 \}$ יש הָסְטַהֲבָהּ הָרֹקֶחֶת.
### Table: Distributions and Moments

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Formula</th>
<th>Expected Value</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binomial ($n, p$)</td>
<td>$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$, $k = 0, \ldots, n$</td>
<td>$np$</td>
<td>$np(1-p)$</td>
</tr>
<tr>
<td>Geometric ($p$)</td>
<td>$p(1-p)^{k-1}$, $k = 1, 2, \ldots$</td>
<td>$\frac{1}{p}$</td>
<td>$\frac{1-p}{p^2}$</td>
</tr>
<tr>
<td>Poisson ($\lambda$)</td>
<td>$e^{-\lambda} \frac{\lambda^k}{k!}$, $k = 0, 1, \ldots$</td>
<td>$\lambda$</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>Uniform ($[a, b]$)</td>
<td>$\frac{1}{b-a}$, $a &lt; x &lt; b$</td>
<td>$\frac{a+b}{2}$</td>
<td>$\frac{(b-a)^2}{12}$</td>
</tr>
<tr>
<td>Exponential ($\lambda$)</td>
<td>$\lambda e^{-\lambda x}$, $x &gt; 0$</td>
<td>$\frac{1}{\lambda}$</td>
<td>$\frac{1}{\lambda^2}$</td>
</tr>
<tr>
<td>Normal ($\mu, \sigma^2$)</td>
<td>$\frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, $x \in \mathbb{R}$</td>
<td>$\mu$</td>
<td>$\sigma^2$</td>
</tr>
</tbody>
</table>

### Theorem 1

If $Z \sim \mathcal{N}(0, 1)$, then

$$
\mathbb{E}[Z^k] = \begin{cases} 
0, & \text{if } k \text{ is odd} \\
1 \cdot 3 \cdot 5 \cdots (k-1), & \text{if } k \text{ is even}
\end{cases}
$$

### Theorem 2

If $X$ is a vector of independent random variables $X = (X_1, \ldots, X_n)^T$ with mean $\mu = (m_1, \ldots, m_n)^T$, then the moment generating function of $X$ is

$$
\phi_X(t) = \exp \left\{ t^T \mu - \frac{1}{2} t^T \Sigma t \right\}
$$

### Theorem 3

If $X$ is a vector of independent random variables $X = (X_1, \ldots, X_n)^T$, then its probability density function is

$$
f_X(x) = \frac{1}{(2\pi)^{n/2} \sqrt{\det \Lambda}} \cdot \exp \left\{ -\frac{1}{2} (x - \mu)^T \Lambda^{-1} (x - \mu) \right\}
$$

### Theorem 4

Covariance $\text{Cov}(X) = \Lambda_{n \times n}$, expected value $\text{E}(X) = \mu$, and $X = (X_1, \ldots, X_n)^T$ mean $\mu = (m_1, \ldots, m_n)^T$. Let

$$
X_1 = (X_1, \ldots, X_k)^T \quad X_2 = (X_{k+1}, \ldots, X_n)^T
$$

$$
m_1 = (m_1, \ldots, m_k)^T \quad m_2 = (m_{k+1}, \ldots, m_n)^T
$$

$$
\Lambda = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{pmatrix}, \quad \Lambda_{11} \in \mathbb{R}^{k \times k}, \Lambda_{22} \in \mathbb{R}^{(n-k) \times (n-k)}
$$
αιר בנבט הווקטור \( X_1 \) הווקטור \( X_2 = x \) המפריד בצורות \( \Sigma \) המניעים נ"ע:

\[
\mu = m_1 + \Lambda_2 \Lambda_2^{-1} (x - m_2)
\]

\[
\Sigma = \Lambda_1 - \Lambda_2 \Lambda_2^{-1} \Lambda_1
\]

**שיטות לנ początku אופטימלי**

משתתפ 1 ריר \( X \) מר' מ来る ירח \( Y = (Y_1, \ldots, Y_n)^T \) הווקטור וקטור

נתוני \( X \) בוים טוית ריבועית מטריצה מטריצה של \( X \)

ואשה \( \hat{X}_{opt}(Y) = \mathbb{E}X + \sum_{i=1}^{n} a_i (Y_i - \mathbb{E}Y_i) \) (3)

\( \mathbb{E} \) סכום המטריצה האופטימלית

כשמ\( \mathbb{a}^T = (a_1, \ldots, a_n) \) טוית טוית:

\( \mathbb{a} = \Lambda_Y^{-1} \cdot \text{Cov}(Y, X) \) (4)

כأمر

\[
\Lambda_Y = \mathbb{E} \left[ (Y - \mathbb{E}Y)(Y - \mathbb{E}Y)^T \right]
\]

\[
\text{Cov}(Y, X) = \mathbb{E} \left[ (Y - \mathbb{E}Y)(X - \mathbb{E}X) \right]
\]

**הוירטuş שימור**

\[
\int_{0}^{\infty} t^k e^{-xt} dt = \frac{k!}{x^{k+1}}, x > 0
\]